6D Muon Ionization Cooling with an Inverse Cyclotron

D. J. Summers[*], S. B. Bracker[*], L. M. Cremaldi[*], R. Godang[*] and R. B. Palmer[†]

[*]*Dept. of Physics and Astronomy, University of Mississippi-Oxford, University, MS 38677 USA*
[†]*Brookhaven National Laboratory, Upton, NY 11973 USA*

Abstract. A large admittance sector cyclotron filled with LiH wedges surrounded by helium or hydrogen gas is explored. Muons are cooled as they spiral adiabatically into a central swarm. As momentum approaches zero, the momentum spread also approaches zero. Long bunch trains coalesce. Energy loss is used to inject the muons into the outer rim of the cyclotron. The density of material in the cyclotron decreases adiabatically with radius. The sector cyclotron magnetic fields are transformed into an azimuthally symmetric magnetic bottle in the center. Helium gas is used to inhibit muonium formation by positive muons. Deuterium gas is used to allow captured negative muons to escape via the muon catalyzed fusion process. The presence of ionized gas in the center may automatically neutralize space charge. When a bunch train has coalesced into a central swarm, it is ejected axially with an electric kicker pulse.

INTRODUCTION

Cooling an ensemble of muons must be completed more rapidly than their 2.2 μs lifetime. Ionization cooling can help [1]. Random muon motion is removed by passage through a low Z material, such as hydrogen, and coherent motion is added with RF acceleration. Designs for 6D muon cooling using linear helical channels [2] at 100 MeV kinetic energies and using frictional cooling [3,4] at keV energies are under investigation. Muon cooling rings have been simulated at various levels [5]. In a ring, the same magnets and RF cavities may be reused each time a muon orbits. Transverse cooling can naturally be exchanged for longitudinal cooling by allowing higher momentum muons to pass through more material. Thus rings cool in all six dimensions.

Small emittance bunches of cold muons are useful to reduce the aperture of the acceleration system for a neutrino factory [6,7] and are required to provide adequate luminosity for a muon collider [8]. At a neutrino factory, accelerated muons are stored in a racetrack to produce neutrino beams ($\mu^- \rightarrow e^- \overline{\nu}_e \nu_\mu$ and $\mu^+ \rightarrow e^+ \nu_e \overline{\nu}_\mu$). Neutrino oscillations have been observed [9] and need more study. Further exploration at a neutrino factory could reveal *CP* violation in the lepton sector [10], and will be particularly useful if the ν_e to ν_τ coupling, θ_{13}, is small [7,11]. A muon collider can do s-channel scans to split the H^0/A^0 Higgs doublet [12]. Above the ILC's 800 GeV there are a large array of supersymmetric particles that might be produced [13] and, if large extra dimensions exist, so could mini black holes [14]. Note that the energy resolution of a 4 TeV muon collider is not smeared by beamstrahlung like CLIC.

OPERATION OF AN AZIMUTHALLY SYMMETRIC INVERSE CYCLOTRON AT LEAR ($\bar{P}$) AND PSI (μ^-)

An inverse cyclotron has been used to slow LEAR anti-protons at CERN [15,16]. An annular quasipotential well, $U(r,z)$, is formed which ferries anti-protons towards the center of an azimuthally symmetric cyclotron. The radius of the annulus decreases with the decreasing angular momentum of the $\bar{p}$.

$$U(r,z) = V(r,z) - (1/(2\eta r^2))(L_g/M + \eta\, rA_\theta)^2, \tag{1}$$

where $\eta = e/M$ and $L_g = L_z - erA_\theta$ is a generalized angular momentum. The radial well deepens with decreasing radius and the vertical well grows shallower (see Fig. 2 of Ref. 15). Particles must adiabatically spiral to the center. If dE/dx is too large, particles will not stay in the magnetic wells. The final $\bar{p}$ swarm has a radius of 1.5 cm, a height of 4 cm, and a kinetic energy of 2 keV. A long bunch train is coalesced into a single swarm, which is roughly the same diameter as the incoming beam. The spiral time is 20 μs with 0.3 mbar hydrogen and about 1 μs with 10 mbar hydrogen. Given the dependence of the cyclotron frequency on mass, $f = \omega/2\pi = qB/2\pi m$, the spiral time for a muon is nine time less than for a $\bar{p}$. The gas pressure in the center must be low, both to allow a particle to spiral all the way in before stopping, and to allow reasonable kicker voltages for axial extraction. An 80 ns electric kicker pulse rising to 500 V/cm in 20 ns is employed. The $\bar{p}$'s move 32 cm in 500 ns. Given that $F = ma$, muons will go nine times farther.

The cyclotron has now been moved from LEAR to PSI where it is used to slow negative muons to a few keV [17]. Three centimeter diameter beams with 30 000 μ^-/s below 50 keV and 0.8 cm diameter beams with 1000 μ^-/s in the 3 to 6 keV kinetic energy range are output for use. A static electric field continuously ejects the muons. The energy absorber and the negatively charged electrode consist of a single 30 μg/cm^2 Formvar foil (polyvinyl formal) with 3 nm of nickel produced by 30 minutes of sputtering.

SKETCH OF A SECTOR INVERSE CYCLOTRON WITH LARGE ADMITTANCE FOR MUONS

A scaling sector cyclotron would allow greater admittance [18] than the azimuthally symmetric cyclotron now running at PSI. For a given $\int \mathbf{B}\cdot d\ell$, the ratio of the fields in the hills and valleys can be adjusted to maximize acceptance. Only radial and neither spiral nor FFAG [19] sectors have been explored so far. The sector cyclotron may be able to function as a damped harmonic oscillator to lower the amplitude of horizontal and vertical betatron motion as a bunch train of muons spirals into a single central swarm.

$$F = \frac{\gamma m v^2}{r} = \frac{qQ}{4\pi\varepsilon_0 r^2} + qvB, \quad r = \frac{\gamma m v^2 \pm \sqrt{(\gamma m)^2 v^4 - 4(qvB)(qQ/4\pi\varepsilon_0)}}{2qvB} \tag{2}$$

With 10^{12} muons in a swarm, space charge is a concern. Table 1 and Eqn. 2 show the effect of space charge. Fortunately, the muons are swarming in an ionized gas which may be able to automatically neuralize the space charge [20]. Electrons experience 200 times
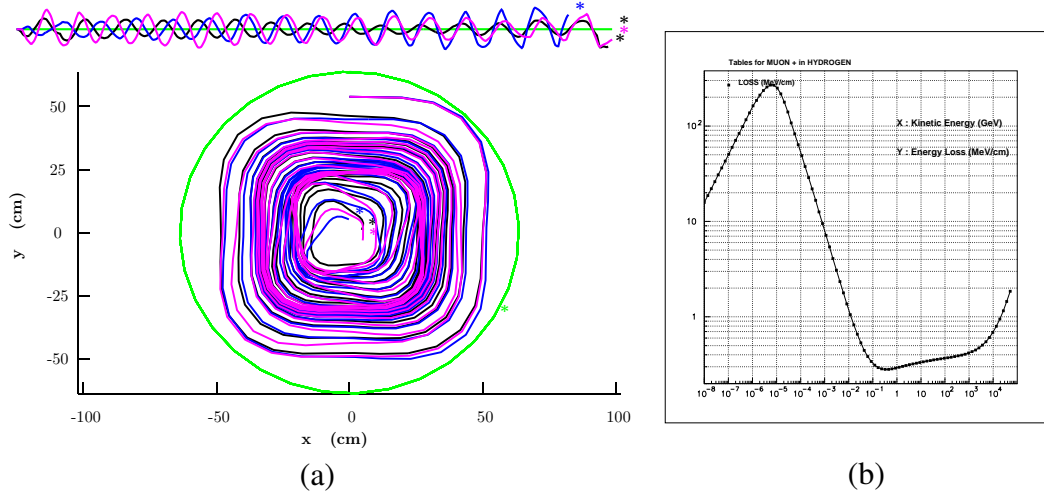
FIGURE 1. **(a)** ICOOL [21] simulation of single turn, energy loss injection. Three identical 172 MeV/c muons are injected into a 1.8 Tesla cyclotron with four sectors and soft edged magnetic fields. The inward spirals differ because of multiple scattering and straggling. The energy loss is caused by radial LiH wedges surrounded by hydrogen gas. The amount of matter encountered in a given orbit decreases adiabatically with radius to allow stable orbits. The upper left trace shows that vertical motion is completely contained within ± 5 cm along the 70 m spiral. The fractional energy loss required in the first turn for injection increases with the width of the muon beam and decreases as the cyclotron's magnetic field is lowered. The injection scaling relation is given by $\Delta p = .3B\Delta r$. Units are GeV/c, Tesla, and meters, respectively. **(b)** Plot of μ^+ energy loss (MeV/cm) in liquid hydrogen versus kinetic energy (GeV) using GEANT3. The default value of "CUTMUO" was decreased from 10 MeV to 10 eV to propagate slow muons. The energy turnover at 8 keV corresponds to a momentum of 1.3 MeV/c. $p = \sqrt{2mE} = \sqrt{2 \times 105.7 \times 0.008}$. Aluminum, copper, iron, and liquid helium show similar results as does the PDG.

the acceleration of muons in an electric field. Movement of 10^{12} electrons in 100 ns requires 1.6 amps of current. A metallic grid might also be used for neutralization.

Muons must spiral in fast enough to minimize decay loss, but must not stop before reaching the central swarm. So the density of the absorber must decrease smoothly with radius. Radial LiH wedges immersed in a gas or high to low pressure gases

TABLE 1. The effect of space charge. Orbital radius of the last muon in millimeters is shown as a function of momentum, magnetic field, and central point charge. The radii come from Eqn. 2 using $v = pc^2/E = pc^2/\sqrt{p^2c^2 + m^2c^4}$. An "$i$" indicates that the radius is partly imaginary.

p	1 Tesla $Q=0$	1 Tesla $Q=10^{12}q$	1 Tesla $Q=4\times10^{12}q$	2 Tesla $Q=0$	2 Tesla $Q=10^{12}q$	2 Tesla $Q=4\times10^{12}q$
16 MeV/c	53	$27+26$	$27+24$	27	$13+13$	$13+11$
8 MeV/c	27	$13+11$	$13+8.7i$	13	$6.7+3.6$	$6.7+9.1i$
4 MeV/c	13	$6.7+9.1i$	$6.7+22i$	6.7	$3.3+7.2i$	$3.3+16i$
2 MeV/c	6.7	$3.3+16i$	$3.3+32i$	3.3	$1.7+11i$	$1.7+22i$
1 MeV/c	3.3	$1.7+22i$	$1.7+45i$	1.7	$.83+16i$	$.83+32i$
.5 MeV/c	1.7	$.83+32i$	$.83+64i$	.83	$.42+23i$	$.42+45i$
.25 MeV/c	.83	$.42+45i$	$.42+90i$	.42	$.21+32i$	$.21+64i$

TABLE 2. Emittance reduction goals for an inverse cyclotron. Emittance goes as $(\Delta p_x \Delta x)(\Delta p_y \Delta y)(\Delta p_z \Delta z)$. A muon collider needs a factor of 10^6 in cooling. The input assumes a factor of 10 in transverse cooling [7]. The output for Δp is from a study of what might be achieved with frictional muon cooling [3].

Δp_x	30 MeV/c $\rightarrow$ 0.3 MeV/c		Δx	70 mm $\rightarrow$ 50 mm
Δp_y	30 MeV/c $\rightarrow$ 0.3 MeV/c		Δy	70 mm $\rightarrow$ 50 mm
Δp_z	30 MeV/c $\rightarrow$ 0.3 MeV/c		Δz	10000 mm $\rightarrow$ 50 mm

separated by beam pipes might meet this criteria. The sector cyclotron geometry must transform into an azimuthally symmetric magnetic bottle as the muons approach the central swarm. Otherwise, as shown by GEANT3, muons will escape though the valleys. In the transition region the field might resemble a hexapole or octupole field as used in an Electron Cyclotron Resonance Ion Source (ECRIS) [22]. If 2×10^{12} 172 MeV/c muons (KE = 96 MeV) arrive at 30 Hz, they will deposit 920 watts of beam power.

Atoms can capture muons. Helium may be used to inhibit muonium ($\mu^+ e^-$) formation [3]. A possibility for negative muons is to use deuterium gas. Muons will catalyze fusion and be freed. The sticking factor is 10%. The reaction appears in Eqn. 3 [23]. 2×10^{12} fusions repeated at 30 Hz only generate 35 watts. The momentum of the freed muon ranges from 0 to 29 MeV/c. A negatively charged absorber foil might also prevent μ^- sticking and is used at PSI. The foil would have to dissipate roughly 100 watts.

$$d + d + \mu^- \rightarrow {}^3He + n + \mu^- + 3.3\,\text{MeV} \;\; \text{or} \;\; t + p + \mu^- + 4.0\,\text{MeV} \tag{3}$$

Busch's theorem (Eqn. 4) [24] has the effect of increasing the emittance as muons leave a magnetic field. A half Tesla field and a 50 mm radius give a 4 MeV/c azimuthal kick. One might be able to use radial iron fins in the exit port to alleviate this effect or reverse and increase the magnitude of the magnetic field to capture the unwanted angular momentum in an absorber after extraction. Using low fields with tall cylindrical swarms that have small diameters works for sure. An RF quadrupole is perhaps a natural choice for acceleration that would immediately follow the extraction electric kicker.

$$\dot{\phi} = [e/(2\pi \gamma m r^2(s))][\Phi(s) - \Phi_k], \quad L_z = xp_y - yp_x = r^2 \gamma m \dot{\phi} = -eBr^2/2 \tag{4}$$

In summary, progress on a large admittance sector cyclotron is underway, including energy loss injection (see Fig. 1a), 6D muon cooling (see Table 2), and an axial electric kicker for extraction. Many thanks to Juan Gallardo and Franz Kottmann for useful suggestions.

REFERENCES

1. A. Skrinsky and V. Parkhomchuk, *Sov. J. Part. Nucl.* **12**, 223 (1981);
   D. Neuffer, *Part. Accel.* **14**, 75 (1983); *Nucl. Instrum. Meth.* **A532**, 26 (2004);
   G. Penn and J. S. Wurtele, *Phys. Rev. Lett.* **85**, 764 (2000);
   K. Kim and C. Wang *Phys. Rev. Lett.* **85**, 760 (2000); *Phys. Rev. Lett.* **88**, 184801 (2002);

G. Franchetti, *Phys. Rev. ST Accel. Beams* **4**, 074001 (2001); G. Dugan, *ibid.,* **4**, 104001 (2001); R. B. Palmer, COOL 05; Y. Derbenev, COOL 05; Y. Torun, COOL 05; M. Cummings, COOL 05; C. Darve, COOL 05; K. Yonehara, COOL 05; K. Beard, COOL 05.

2. Y. Derbenev, R. Johnson, *Nucl. Instrum. Meth.* **A532**, 470 (2004); *AIP Conf. Proc.* **671**, 328 (2003); *Phys. Rev. ST Accel. Beams* **8**, 041002 (2005); R. Johnson, COOL 05.

3. H. Abramowicz *et al., Nucl. Instrum. Meth.* **A546**, 356 (2005).

4. R. Galea *et al., Nucl. Instrum. Meth.* **A524**, 27 (2004); *J. Phys.* **G29**, 1653 (2003); A. Caldwell, *J. Phys.* **G29**, 1569 (2003); D. Taqqu, *AIP Conf. Proc.* **372**, 301 (1996); M. Muhlbauer *et al., Nucl. Phys. Proc. Suppl.* **51A**, 135 (1996); *Hyperfine Interact.* **119**, 309 (1995).

5. V. I. Balbekov and A. Van Ginneken, *AIP Conf. Proc.* **441**, 310 (1998); J. S. Berg, R. C. Fernow, and R. B. Palmer, *J. Phys.* **G29**, 1657 (2003); R. B. Palmer, *J. Phys.* **G29**, 1577 (2003); *Nucl. Instrum. Meth.* **A532**, 255 (2004); R. Palmer *et al., Phys. Rev. ST Accel. Beams* **8**, 061003 (2005).

6. A. Blondel *et al., Nucl. Instrum. Meth.* **A451**, 102 (2000); R. Palmer *et al., ibid.,* **A451**, 265 (2000); D. Ayres *et al.*, physics/9911009; N. Holtkamp *et al.*, "A neutrino source based on a muon storage ring," Fermilab-Pub-00-108-E (2000); S. Ozaki *et al.*, "Study II," BNL-52623 (2001); M. Yoshida, *Nucl. Phys. Proc. Suppl.* **149**, 94 (2005); D. Kaplan, COOL 05.

7. C. Albright *et al.,* physics/0411123.

8. G. Budker, *AIP Conf. Proc.* **352**, 4 (1996); *ibid.,* **352**, 5 (1996); A. Skrinsky, *ibid.,* **352**, 6 (1996); D. Neuffer, *AIP Conf. Proc.* **156**, 201 (1987); *Nucl. Instrum. Meth.* **A350**, 27 (1996); R. Palmer *et al., Nucl. Phys. Proc. Suppl.* **51A**, 61 (1996); *AIP Conf. Proc.* **372**, 3 (1996); R. B. Palmer, J. C. Gallardo, A. Tollestrup, and A. Sessler, *Sci. Cult.* **13**, 39 (1998); R. Raja, A. Tollestrup, *Phys. Rev.* **D58**, 013005 (1998); C. Ankenbrandt *et al., Phys. Rev. ST Accel. Beams* **2**, 081001 (1999); M. Alsharo'a *et al.*, *Phys. Rev. ST Accel. Beams* **6**, 081001 (2003).

9. R. Davis *et al.* (Homestake), *Phys. Rev. Lett.* **20**, 1205 (1968); *Astrophys. J.* **496**, 505 (1998); Y. Fukuda *et al.* (SuperK), *Phys. Rev. Lett.* **81**, 1562 (1998); *Phys. Rev. Lett,* **93**, 101801 (2004); Q. R. Ahmad *et al.* (SNO), *Phys. Rev. Lett.* **89**, 011301 (2002); *Phys. Rev. Lett.,* **92**, 181301 (2004); K. Eguchi *et al.* (KamLAND), *Phys. Rev. Lett.* **90**, 021802 (2003); *ibid.* **94**, 081801 (2005); M. Ahn *et al.* (K2K), *Phy. Rev. Lett.* **90**, 041801 (2003); E. Aliu, *ibid.* **94**, 081802 (2005).

10. S. Geer, *Phys. Rev.* **D57**, 6989 (1998); C. Albright *et al.,* hep-ex/0008064; V. Barger *et al., Phys. Rev. Lett.* **45**, 2084 (1980); A. Cervera *et al., Nucl. Phys.* **B579**, 17 (2000).

11. M. Maltoni *et al., New J. Phys.* **6**, 122 (2004).

12. V. Barger *et al., Phys. Rev. Lett.* **75**, 1462 (1995); *Phys. Rept.* **286**, 1 (1997); D. Atwood and A. Soni, *Phys. Rev.* **D52**, 6271 (1995); J. F. Gunion, hep-ph/9802258.

13. J. Ellis, LCWS 04, hep-ph/0409140.

14. R. Godang *et al.,* hep-ph/0411248; M. Cavaglia and S. Das, *Class. Quant. Grav.* **21**, 4511 (2004).

15. J. Eades and L. M. Simons, *Nucl. Instrum. Meth.* **A278**, 368 (1989).

16. J. Eades *et al., Nucl.Phys.Proc.Suppl.* **8**, 457 (1989); E. Aschenauer *et al., Sov. J. Nucl. Phys.* **55**, 856 (1992); L. M. Simons *et al., Springer Proc. Phys.* **59**, 33 (1992); *Phys. Scripta* **T22**, 90 (1988); *Hyperfine Interact.* **81**, 253 (1993); *Phys. Bl.* **48**, 261 (1992); *Nucl. Instrum. Meth.* **B87**, 293 (1994); D. Horváth *et al., Nucl. Instrum. Meth.* **B85**, 736 (1994).

17. P. DeCecco *et al., Nucl. Instrum. Meth.* **A394**, 287 (1997); Franz.Kottmann@psi.ch.

18. A. Garren *et al.*, "Gas Filled Rings for Muon Beam Cooling," COOL 05; A. A. Garren *et al., AIP Conf. Proc.* **297**, 403 (1994); D. J. Summers *et al.*, Int. J. Mod. Phys. **A20**, 3851 (2005).

19. M. Craddock, *CERN Cour.* **44N6**, 23 (2004); D. J. Summers, *Int. J. Mod. Phys.* **A20**, 3861 (2005); K. Symon, D. Kerst, L. Jones, L. Laslett, and K. Terwilliger, *Phys. Rev.* **103**, 1837 (1956); J. S. Berg, *AIP Conf. Proc.* **642**, 213 (2003); D. Trbojevic *et al., AIP Conf. Proc.* **530**, 333 (2000); S. Koscielniak, C. Johnstone, *AIP Conf. Proc.* **721**, 467 (2004); *NIM* **A523**, 25 (2004); E. Keil and A. Sessler, *NIM* **A538**, 159 (2005); Y. Mori, *ICFA Beam Dyn. Newslett.* **29**, 20 (2002).

20. Lloyd P. Smith, W. E. Parkins, and A. T. Forrester, "On Separation of Isotopes in Quantity by Electromagnetic Means," Phys. Rev. **72**, 989 (1947); C. Niemann *et al.*, J. Phys. **D36**, 2102 (2003).

21. R. C. Fernow, *AIP Conf. Proc.* **721**, 90 (2004); PAC99, eConf C990329, THP31.

22. A. Girard *et al., Rev. Sci. Instrum.* **75**, 1381 (2004); A. Zelenski *et al., ibid.* **75**, 1535 (2004).

23. L. I. Ponomarev, Contemp. Phys. **31**, 219 (1990); P. Strasser *et al., Phys. Lett.* **B368**, 32 (1996).

24. H. Busch, *Annalen Phys.* **81**, 974 (1926); A. W. Chao and M. Tigner, *Handbook of Accelerator Physics and Engineering,* page 101 (1999).